MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 1, 2019/2020

EME4076 – MECHANICAL VIBRATIONS

(ME)

23 OCTOBER 2019
2.30 p.m. – 4.30 p.m.
2 Hours

INSTRUCTIONS TO STUDENT

1. This Question paper consists of 9 pages including cover page with 5 Questions and 3 appendices.

2. Attempt **FOUR** out of **FIVE** questions. All questions carry equal marks and the distribution of the marks for each question is given.

3. Please write all your answers in the Answer Booklet provided.

Question 1

Figure Q1 shows a crude oil pumping system which consists of a motor-driven walking beam and sucker rod attached to a mechanical lift pump. The diameter of the well pipe is 51 mm. The steel sucker rod is 19 mm in diameter and 1905 m long. The motor of the pumping system is operated at a speed of 18 rpm. The motion of the upper end of the sucker rod at A is essentially sinusoidal with a stroke of 1.25 m. During the downward stroke a valve at the lower end of well pipe opens to let a quantity of oil into the well pipe. This column of oil in the well pipe is then lifted during the upward stroke by the piston at the end of the sucker rod and discharged into the connecting pipe at B. The equivalent mass of the sucker rod immersed in the column of oil and the column of oil lifted at each stroke of the pump is 6850 kg. The Young's modulus of the steel is given as $E = 2.1 \times 10^{11}$ N/m^2 and the oil has a damping ratio ς of 0.5.

(a) Derive the equation of motion of the oil pumping in terms of x where x is the displacement of the piston.

[10 marks]

(b) Determine the stroke of the piston and the output of the well in m^3/min, (i) if the sucker rod is considered as extensible, and (ii) if the sucker rod is considered as rigid.

[15 marks]

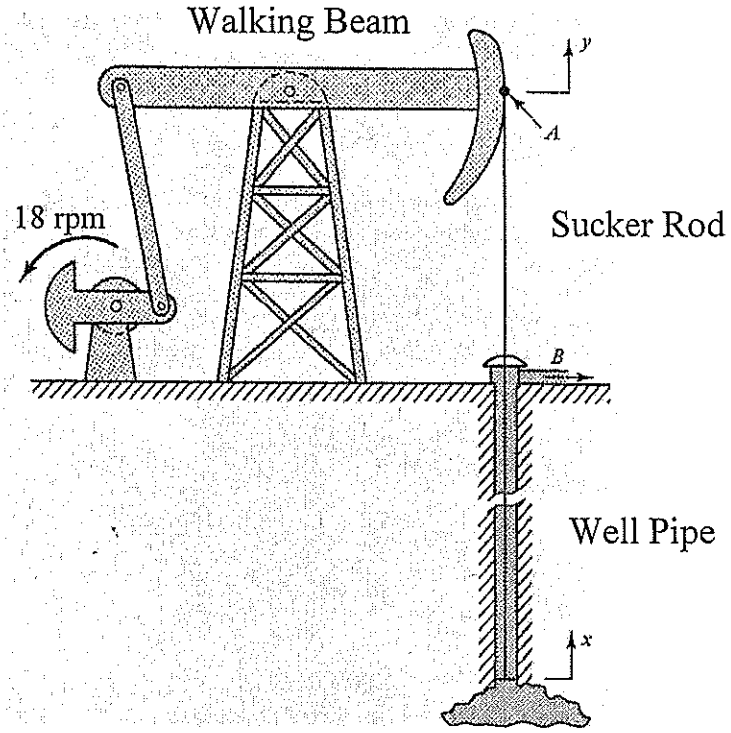


Figure Q1

Continued .....

Question 2

(a) Figure Q2(a) shows the load-deflection curve of two types of cylinder rubber isolator, A and B, with damping ratio ζ of 0.02. The maximum safe static load for each isolator is marked with a dash on the graph. A 40-kg instrument is used on a table which vibrates with a frequency of 1100 rpm due to nearby machinery. Determine whether or not the isolator whose load-deflection curve shown in Figure Q2(a) can be used at each of the four corners of the instrument so that no more than 20% of the table motion is transmitted to the instrument. Determine how much mass, if any, is required to add to the instrument such that the system is operated at the maximum safe static load of the isolator that you choose.

[13 marks]

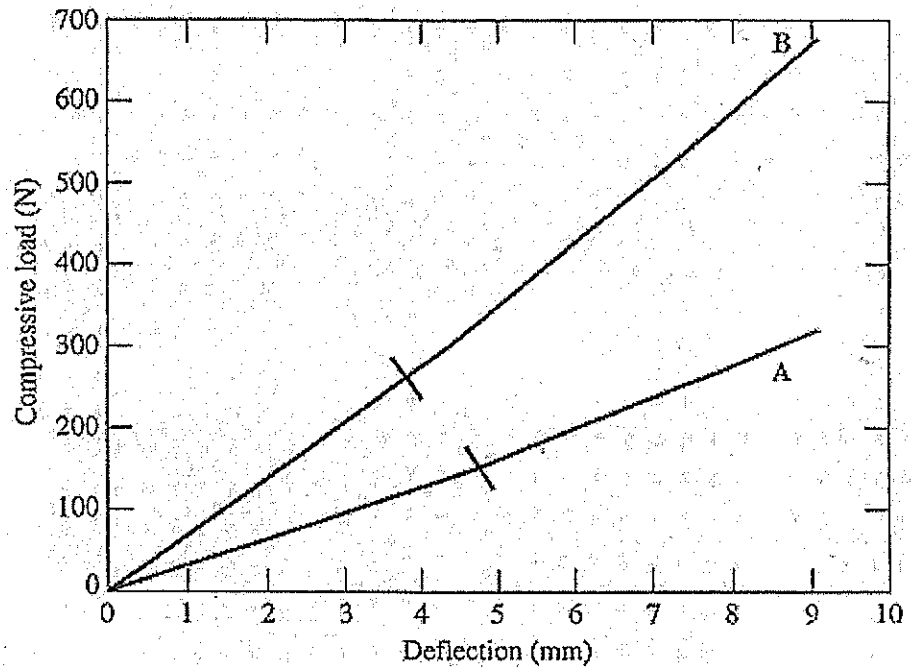


Figure Q2(a)

(b) The rotor of a specific machine has a total mass of 1000 kg and a measured unbalance of $m_0 e = 0.22$ kg-m. The machine is operated at a speed of 500 rpm. Design an isolator (pick k and c) such that it is able to reduce the amplitude of the force exerted on the floor to 1% of the force generated at the operating speed, and also is able to limit the maximum force transmitted to the floor at resonance to the amount of never greater than two times of the generated force. With this isolator, determine the maximum displacement of the machine at the operating speed.

[12 marks]

Continued .....

Question 3

Figure Q3 shows a SUV with mass M used a bumper hitch to pull a trailer with mass m_1. The hitch acts as an elastic spring with stiffness k. Inside the trailer, there is a lighting fixture attached to the ceiling of the trailer. The length of the cable of the fixture is l and the mass of the bulb and its bracket is m_2. Due to unforeseen circumstances, the SUV is suddenly stopped and at the same time the trailer is hit by an external sinusoidal force of $F = F_0 \sin\omega t$. This cause the trailer and the lighting feature to vibrate.

(a) Derive the linear differential equations of the system in terms of x_1 and θ where x_1 and θ are the displacement of the trailer and angular rotation of the lighting feature, respectively. Since the mass of the SUV is much larger than the trailer and the lighting feature, the SUV can be treated as a fixed support.

[12 marks]

(b) Find the steady-state response of the trailer and the lighting feature.

[8 marks]

(c) Since $x_2 = x_1 + l\theta$ where x_2 is the displacement of the lighting feature, show that the vertical distance h from the mass m_2 of the bulb and its bracket to the node of the cable is given by the relation $h = l(\omega_n/\omega)^2$ where $\omega_n = \sqrt{g/l}$.
Hint: the node will occur when x_1 and x_2 are opposite to each other.

[5 marks]

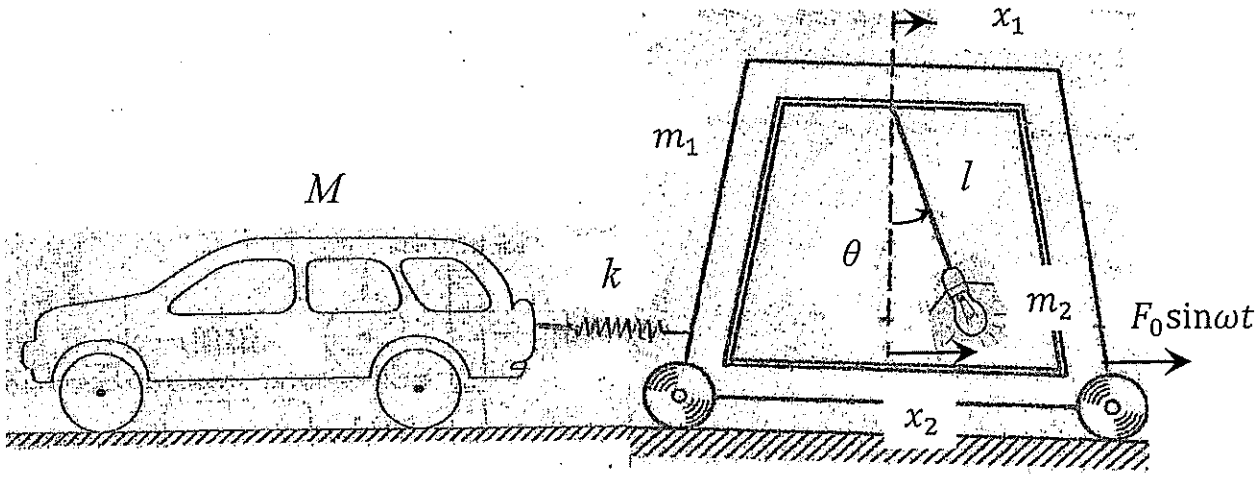


Figure Q3

Continued .....

Question 4

Figure Q4 shows a horizontal antenna consists of 1-m long aluminium cantilever beam with a 10 mm × 10 mm square cross section. A vulture with a mass of 1 kg after long searching for food clings to the free end of the antenna with a vertical velocity of 1.2 m/s. This causes the antenna to vibrate.

(a) It is observed that the amplitude of the sixth oscillation of the antenna is half of the amplitude of the fourth oscillation. Assume antenna is massless. Find the response of the system. The Young's modulus of aluminium is 70 GPa.

[13 marks]

(b) What is the maximum displacement of the antenna from equilibrium position? How long does it take for this to happen?

[8 marks]

(c) If vertical distance from the roof of the house to the horizontal position of the antenna is 30 cm, will the free end of the antenna hit the roof once the vulture clings to it? Comment your answer.

[4 marks]

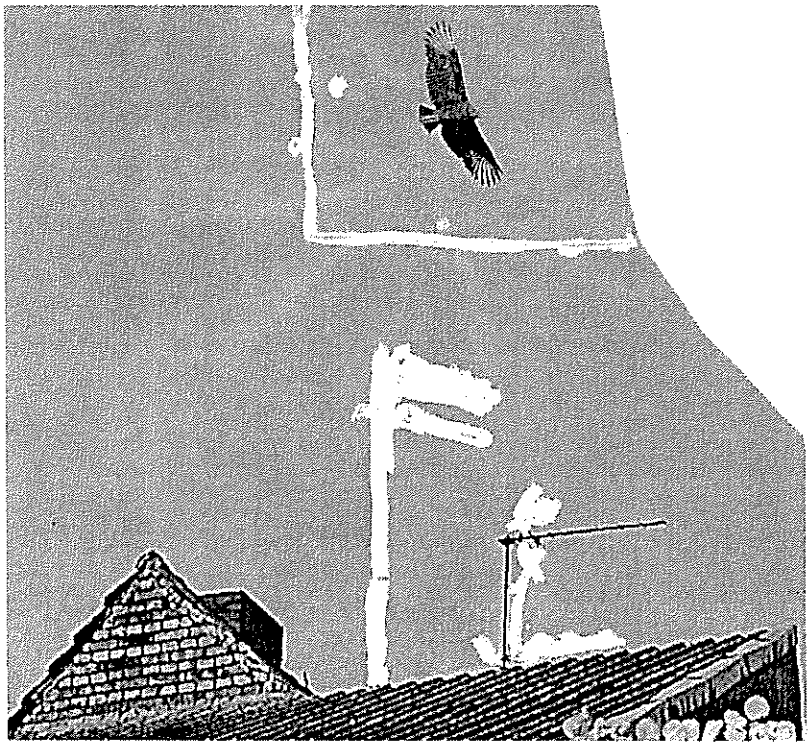


Figure Q4

Continued .....

Question 5

Figure Q5(i) shows a speed bump in a residential area. It is used to force drivers to slow down. Figure Q5(ii) shows a car, which can be modeled as a spring-mass system, is going over the speed bump. The car is traveling at a constant speed of v. The bump can be modeled as a half sine which has the form of $y(s) = Y\sin(\pi s/\delta)$ where s, δ and Y are the horizontal distance variable, the length and the height of the bump, respectively.

(a) Show that the equations of motion of the car and of the bump in terms of time t can respectively be expressed as

$$m\ddot{z} + kz = -m\ddot{y} \quad \text{and} \quad y(t) = \begin{Bmatrix} Y\sin(\pi t/t_0) & 0 \le t \le t_0 \\ 0 & t > t_0 \end{Bmatrix}$$

where t_0 is the time for the car to finish going over the bump and $z = x - y$, x and y are the displacements of the car and the bump, respectively.

[6 marks]

(b) Show that relative response z of the car for the time $0 \le t \le t_0$ is

$$z = \frac{Y\omega_b^2}{\omega_n}\left[\frac{\omega_b}{\left(\omega_b^2 - \omega_n^2\right)}\right]\left\{\sin\omega_n t - \frac{\omega_n}{\omega_b}\sin\omega_b t\right\} \text{ where } \omega_b = \frac{\pi v}{\delta}$$

Hint: use identity $\sin(a)\sin(b) = (1/2)\cos(a-b) - (1/2)\cos(a+b)$.

[13 marks]

(c) Show that the maximum relative deflections of the car when the car is traveling over the bump at high speed $(\omega_b \gg \omega_n)$ and low speed $(\omega_b \ll \omega_n)$ are

$$z_{\max} = \frac{Y\omega_b^3}{\omega_n\left(\omega_b^2 - \omega_n^2\right)} \quad \omega_b \gg \omega_n, \text{ and } z_{\max} = \frac{Y\omega_b^2}{\omega_n^2 - \omega_b^2} \quad \omega_b \ll \omega_n, \text{ respectively.}$$

[6 marks]

Figure Q5(i)

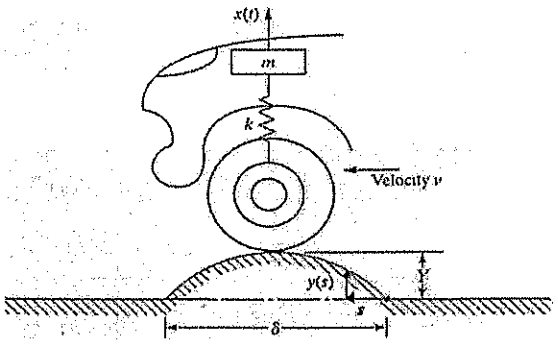


Figure Q5(ii)

Continued .....

Appendix 1

Vibrations

Equations of Motion

$$m\ddot{x} + c\dot{x} + kx = F_0 \sin \omega t$$

General Solution

$$x(t) = x_c(t) + x_p(t)$$

$$x_p(t) = D \sin(\omega t - \varphi)$$

$$D = \frac{F_0/k}{\sqrt{[1-r^2]^2 + (2\zeta r)^2}} \qquad where \quad r = \frac{\omega}{\omega_n}$$

$$\varphi = \tan^{-1} \frac{2\zeta r}{1-r^2}$$

$$x_c(t) = D_1 e^{\lambda_1 t} + D_2 e^{\lambda_2 t} \qquad \zeta > 1$$

$$x_c(t) = (B + Ct) e^{-\omega_n t} \qquad \zeta = 1$$

$$x_c(t) = e^{-\zeta \omega_n t} (B \cos \omega_d t + C \sin \omega_d t) \qquad \zeta < 1$$

$$\omega_n = \sqrt{k/m} = 2\pi f_n, \qquad f_n = \frac{1}{\tau_n} = \frac{\omega_n}{2\pi}$$

$$\zeta = \frac{c}{2m\omega_n}$$

$$\lambda_{1,2} = -\zeta \omega_n \pm \omega_n \sqrt{\zeta^2 - 1} \qquad \zeta \geq 1$$

$$\omega_d = \omega_n \sqrt{1-\zeta^2} \qquad \zeta < 1$$

Force and Displacement Transmissibility

$$\frac{F_T}{F_0} = \frac{X}{Y} = \left[\frac{1 + (2\zeta r)^2}{(1-r^2)^2 + (2\zeta r)^2}\right]^{\frac{1}{2}}$$

Rotating Unbalance: Amplitude of Response

$$X = \frac{m_0 e}{m + m_0} \frac{r^2}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}}$$

Fourier's series

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} [a_n \cos n\omega t + b_n \sin n\omega t]$$

$$a_n = \frac{2}{\tau}\int_0^{\tau} x(t) \cos n\omega t \, dt \qquad b_n = \frac{2}{\tau}\int_0^{\tau} x(t) \sin n\omega t \, dt$$

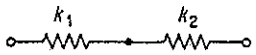
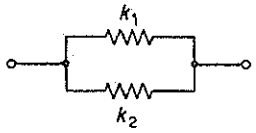
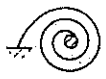
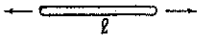
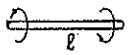
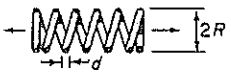
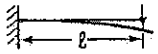
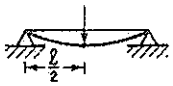
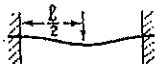
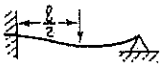
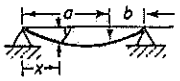
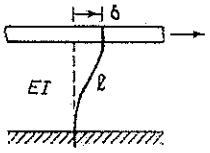
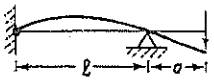
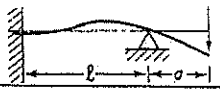
Unit response function (underdamped)

$$h(t-\tau) = \frac{e^{-\zeta\omega_n(t-\tau)}}{m\omega_d} \sin \omega_d (t-\tau)$$

Continued .....

Appendix 2

Table of Spring Stiffness

	$k = \dfrac{1}{1/k_1 + 1/k_2}$	
	$k = k_1 + k_2$	
	$k = \dfrac{EI}{l}$	I = moment of inertia of cross-sectional area l = total length
	$k = \dfrac{EA}{l}$	A = cross-sectional area
	$k = \dfrac{GJ}{l}$	J = torsion constant of cross section
	$k = \dfrac{Gd^4}{64nR^3}$	n = number of turns
	$k = \dfrac{3EI}{l^3}$	k at position of load
	$k = \dfrac{48EI}{l^3}$	
	$k = \dfrac{192EI}{l^3}$	
	$k = \dfrac{768EI}{7l^3}$	
	$k = \dfrac{3EIl}{a^2b^2}$	$y_x = \dfrac{Pbx}{6EIl}(l^2 - x^2 - b^2)$
	$k = \dfrac{12EI}{l^3}$	
	$k = \dfrac{3EI}{(l + a)a^2}$	
	$k = \dfrac{24EI}{a^2(3l + 8a)}$	

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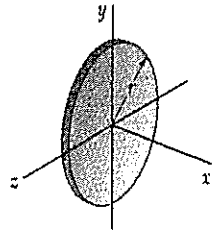
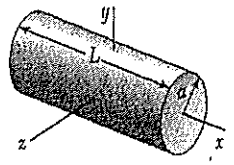
Appendix 3: Mass Moment of Inertia of Homogenous Solids



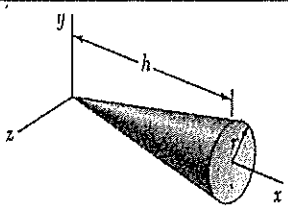
$I_z = \frac{1}{2}mr^2$

$I_y = I_z = \frac{1}{4}mr^2$



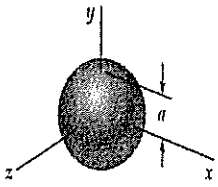
$I_x = \frac{1}{2}ma^2$

$I_y = I_z = \frac{1}{12}m(3a^2 + L^2)$

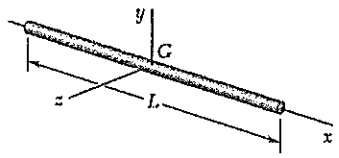


$I_x = \frac{3}{10}ma^2$

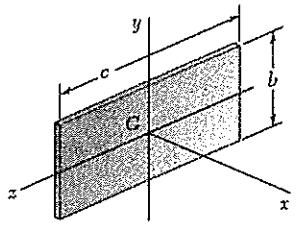
$I_y = I_z = \frac{3}{5}m(\frac{1}{4}a^2 + h^2)$



$I_x = I_y = I_z = \frac{2}{5}ma^2$



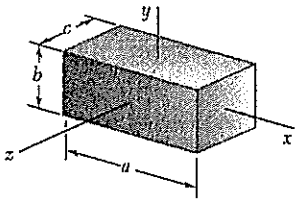
$I_y = I_z = \frac{1}{12}mL^2$



$I_x = \frac{1}{12}m(b^2 + c^2)$

$I_y = \frac{1}{12}mc^2$

$I_z = \frac{1}{12}mb^2$



$I_x = \frac{1}{12}m(b^2 + c^2)$

$I_y = \frac{1}{12}m(c^2 + a^2)$

$I_z = \frac{1}{12}m(a^2 + b^2)$

End of Paper